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SURFACE DETAIL AND BACKSCATTER FROM COHERENTLY ILLUMINATED TARG--ETC(U)
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TECHNICAL REPORT TR-77-3

SURFACE DETAIL AND BACKSCATTER FROM COHERENTLY ILLUMINATED TARGETS ROTATING ABOUT THE AXIS OF SYMMETRY

Physical Sciences Directorate
Technology Laboratory

1 February 1977

U.S. ARMY MISSILE RESEARCH AND DEVELOPMENT COMMAND

Redstone Arsenal, Alabama 35809

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A theoretical analysis of the ratio of rms backscattered intensity fluctuation to average intensity for a coherently illuminated target rotating about an axis of symmetry is presented in this report. The ratio $(\delta I)_{rms}/\langle I \rangle$ depends on the rms random phase fluctuation and the number of decorrelation area cells on the exposed target surface. For the case where the random phase fluctuations are due to surface roughness, the ratio depends on the equivalent

ABSTRACT (Continued)

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delta I sub Rms

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surface detail parameters: (1) rms height fluctuation and (2) the product of average roughness slope and the square root of the exposed target surface. The ratio $(\delta I)_{rms}/\langle I \rangle$ is also dependent on the illumination wavelength, and the two surface detail parameters can be uniquely determined if the wavelength dependence of the ratio is measured. The analysis presented is especially relevant to laser radar applications.

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SURFACE DETAIL AND BACKSCATTER FROM COHERENTLY ILLUMINATED TARGETS ROTATING ABOUT THE AXIS OF SYMMETRY

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CONTENTS

I. INTRODUCTION

When a distant target is illuminated by a coherent light source, the far-field backscatter intensity is speckled. In general, there is a constant component and a spatially varying component. Hence, the intensity measured by a backscatter detector (at the illumination site) fluctuates as the target body turns. If the target is a flat surface, a discrete Fourier transform (DFT) of the backscatter intensity indicates the decorrelation length for the random phase variations of the reflected light at the target surface (due to surface roughness, for example [1, 2]). If, however, the body is curved in such a way that the surface always moves through the same surface-of-curvature (e.g., a cone, cylinder, or sphere rotating about the axis-of-symmetry), then the DFT will only yield information concerning the target width, provided its surface is rough [3].

It is for this class of targets that the present formulation is developed, and it is pertinent to monostatic laser radar applications. The ratio of rms intensity fluctuation to average intensity is formed from the measured data. This ratio is dependent upon the rms random phase fluctuation of reflected light at the target surface and the number of decorrelation cells on that surface. For the case where the phase fluctuation is due to surface roughness, the equivalent parameters are: (1) rms height fluctuation and (2) the product of average roughness slope and the square root of the exposed target surface. Both of these parameters can be uniquely determined if measurements are made as a function illumination wavelength.

Figure 1 illustrates the measurement configuration. The angle α is the orientation of the turning surface at any instant. In the model presented in this treatment, the target's surface field is assumed to vary from the average in a half-cycle sinusoid within each decorrelation area cell. The amplitude and period for each cell is independent of that in any other. The effective phase for each cell is random with equal probability between the bounds $-\phi_m$ and ϕ_m. The phase fluctuation statistics are assumed to be the same for all directions over the body. The decorrelation area $\Delta s_c = \ell_c^2$ defined here relates only to fluctuations of the reflected surface field from the average. It is further assumed that the number of decorrelation cells on the body is large.

Polarization is not explicitly treated in this report. If the polarization effect relates to the surface curvature (from the line of observation), forming the ratio of rms intensity fluctuation to average intensity cancels out its effect.

The Doppler effect due to target rotation is not included in this treatment. Although the far-field intensity due to a given point on

the surface is critically dependent upon the Doppler shift, there is a compensating effect in the integration of the effect due to the whole surface which greatly reduces its effect on the overall statistics of the far-field. Thus, the treatment presented here is valuable despite the exclusion of the Doppler effect. This, however, does not imply that the Doppler effect is not important to the spread in the frequency spectrum of the reflected light.

The present work is intended to be a guide to the understanding and development of laser radar applications.

II. THEORY

A. General Formulation

A target body which rotates so that its surface always passes through the same surface-of-curvature is considered. Examples may be a sphere, cone, or cylinder rotating about their axes of symmetry. The axis is assumed to be approximately perpendicular to the line of observation. The distant target is illuminated with coherent light, and the far-field backscatter is detected at the illumination site. The electric field amplitude at the detector is

$$A = A_a + \Delta A \quad , \tag{1}$$

where A_a is the average amplitude and ΔA is the fluctuation from the average which occurs as the target rotates. Thus, a speckle field is constantly sweeping across the detector. According to the Rayleigh-Sommerfeld diffraction formulation [4], A is related to the surface field a of the target by

$$A = \frac{\exp jkR}{j\lambda R} \int_S a \exp[jk(r - R)] \cos(\underline{n},\underline{r}) \, ds \quad , \tag{2}$$

where R is the distance from the detector to the nearest point of the target surface, r is the distance to the surface at any given point (average surface position at this site if the surface is rough), and $\cos(\underline{n},\underline{r})$ is the angle between the surface normal and the line of observation (Figure 1). The integration is over the exposed surface. It is assumed that the integrand of Equation (2) can be expressed as

$$a_1(\underline{r}) \, F(\underline{n},\underline{r}) \quad ,$$

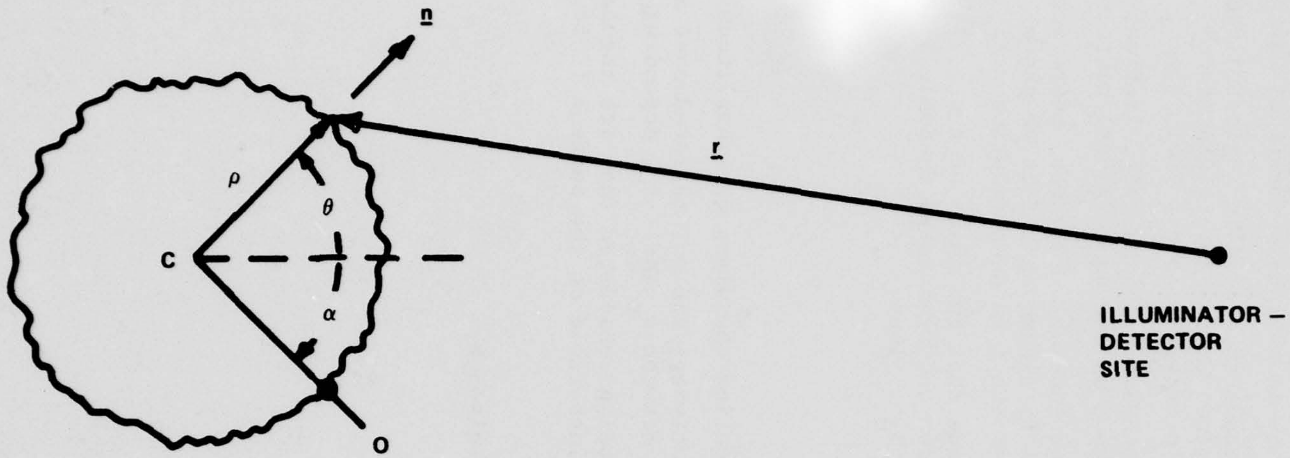

Figure 1. Detection of far-field backscatter. NOTE: It is assumed that $\underline{r} >> \rho$. The quantity α is the reference angle between the observation line and the line $\overline{co}$ which is fixed to the surface. As the body rotates, α changes.

where $F(\underline{n},\underline{r})$ is $\cos(\underline{n},\underline{r})$ times a phase shift factor related to surface curvature (a phase component which is due to the fact that different parts of the target surface are not equidistant from the plane of the incident illumination field). The a_1 is the field amplitude which would exist at the surface if the curvature could be neglected and thus only contains a phase factor determined by the detailed surface properties such as roughness, dipole irregularities, etc. The a_1 may depend on the angle $\underline{n},\underline{r}$ in some cases. For example, if fluctuations in a_1 are due to roughness, then an exponent $2k\ h(r)\ \cos(\underline{n},\underline{r})$ is involved (h = height from average surface). However, there are compensating aspects that allow one to assume an effective phase change due to roughness of $2k\ h_i$ for the i th decorrelation cell (Appendix A). The assumption is made that the phase of the cells does not depend on the surface orientation and fluctuates randomly with equal probability between ϕ_m and $-\phi_m$. Thus,

$$a_1 = a_o\ p\ \exp j\phi \quad , \tag{3}$$

where a_o is a real constant representing the plane wave amplitude of the incident field, p is the reflectivity, and ϕ integrated over a decorrelation cell varies randomly between ϕ_m and $-\phi_m$, depending only on the region of the surface being considered (not its angular orientation). Thus, a_1 is the surface field on the surface if it were flat and normally illuminated.

The terms of Equation (1) are given by

$$A_a = \frac{1}{j\lambda R} \exp jkr \int a_{1a}\ F\ ds \tag{4}$$

and

$$\Delta A = \frac{1}{j\lambda r} \exp jkR \int \Delta a_1\ F\ ds \quad , \tag{5}$$

where

$$a_{1a} = \langle a_1 \rangle \quad , \tag{6}$$

and

$$\Delta a_1 = a_1 - a_{1a} \quad . \tag{7}$$

The quantity Δa_1 fluctuates as a random function of α. It is seen that

$$\int a_{1a} F \, ds = a_{1a} \int F \, ds \quad , \tag{8}$$

since a_{1a} is a constant over the surface.

The decorrelation cell is defined as the constructed surface area for which a_1 locally departs from a_{1a}, either positively or negatively (Figure 2). Thus, the decorrelation cell area is not necessarily proportional to the square of the correlation length based on an autocorrelation of ϕ. Such irregularity in the behavior of $\phi(\alpha)$ exists that a correlation of Δa_1 with itself will fall off to an average of zero for displacement of $<\ell_i> = \ell_c$. Even if Δa_1 is varying sinusoidally [with higher frequency than $\phi(\ell)$] because $\lambda < k\ \phi_m$ ($\phi_m = \phi_{max}$), the derivative $d\phi/d\ell$ is assumed to vary sufficiently that the period of the sinusoid fluctuates from one-half cycle of Δa_1 to the next so that the correlation function reaches its first zero at about $\ell = \ell_c$ and then oscillates randomly with diminishing amplitude. Thus, each "peak" or "valley" of Δa_1 is held to be independent of any other.

It is important to realize that the decorrelation area defined here refers to random fluctuations in Δa_1, however small compared to a_{1a}, and not phase or a_1 itself. A reordering of the integration in Equation (5) is performed such that Δa_1 runs smoothly from the highest to the lowest value, and F fluctuates randomly about its average. The number of cells N is taken to be sufficiently large that the fluctuating F integrated over an incremental range of Δa_1 (small compared to the maximum value of Δa_1) is the same as the overall average. This allows one to write*

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*To be absolutely rigorous, the integral over Δa_1 is not independent of F. However, the "average rms fluctuation" of $\int \Delta a_1 F \, ds/<F>$ as α varies is the same as that for the integral over Δa_1 alone. Since this treatment leads toward measurements of averages, Equation (9) may be assumed without error.

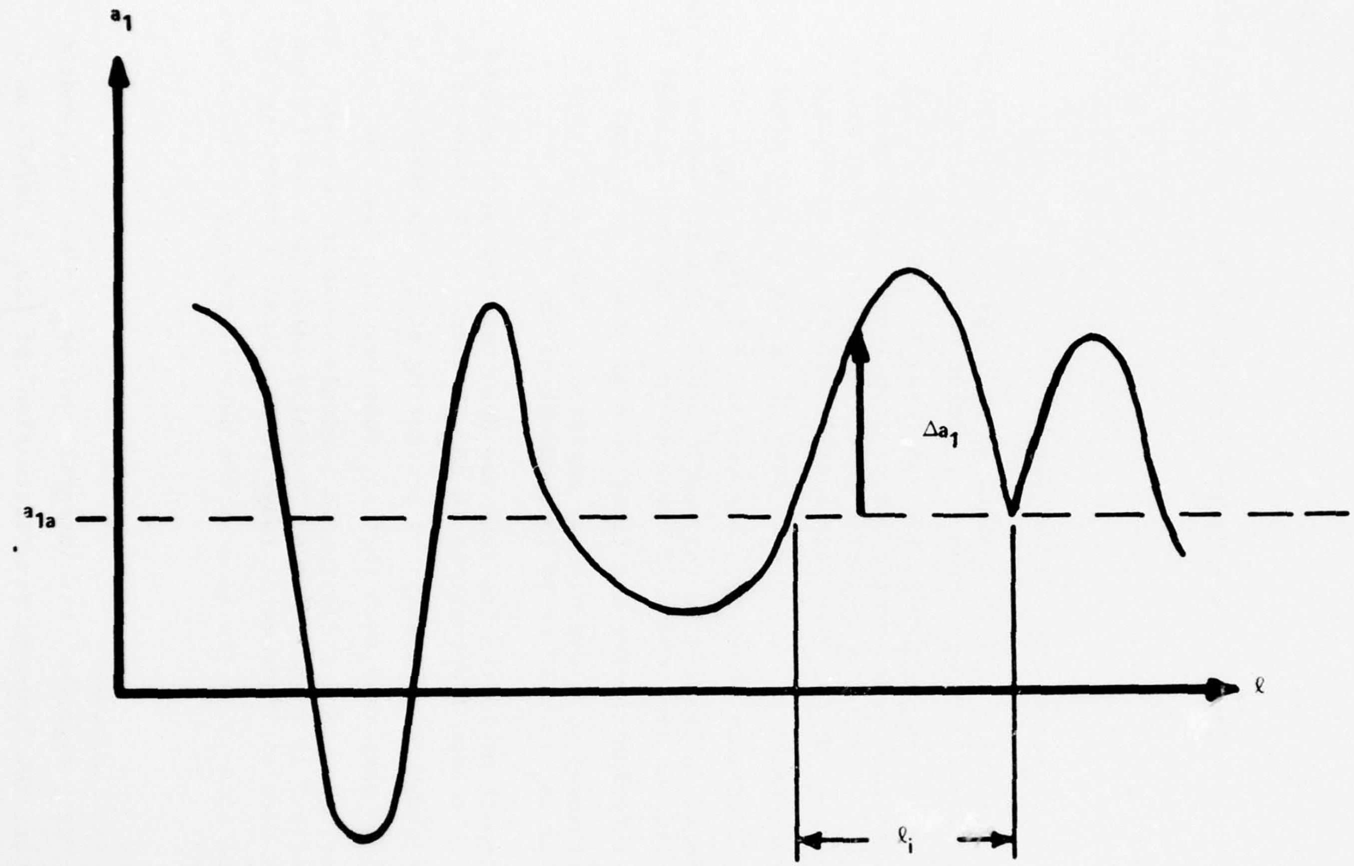

Figure 2. Variation of a_1 along the constructed surface. NOTE: The figure represents a profile of the field along a line ℓ on the surface. The length $\ell_i \approx \sqrt{\Delta s_i}$ is the decorrelation cell length.

$$\int \Delta a_1 \; F \; ds \simeq \langle F \rangle \int \Delta a_1 \; ds \quad , \tag{9}$$

where

$$\langle F \rangle = \frac{1}{S} \int F \; ds \quad . \tag{10}$$

S is the total exposed area of the target. A clarification of Equation (9) is shown in Figure 3.

Note that

$$\int \Delta a_1 \; ds \equiv \sum_{i=1}^{N} \Delta a_{1i} \; \Delta s_i \quad , \tag{11}$$

where Δa_{1i} is the average value of Δa_1 in the i th member of the N cells and Δs_i is the correlation area of that cell. The area Δs_i fluctuates independently of Δa_{ij} about an average value Δs_c. Hence,

$$\int \Delta a_1 \; ds \simeq \Delta s_c \; \delta\Delta a_1 \equiv \Delta s_c \sum_{i=1}^{N} \Delta a_{1i} \quad . \tag{12}$$

or, using Equations (9) and (10),

$$\int F \; \Delta a_1 \; ds \simeq \int F \; ds \; \frac{\Delta s_c}{S} \; \delta\Delta a_1 \quad , \tag{13}$$

where

$$\delta\Delta a_1 \equiv \sum_{i=1}^{N} \Delta a_{1i} \quad , \tag{14}$$

and

$$\Delta s_c / S = N \quad . \tag{15}$$

Thus, using Equations (4), (5), (8), (9), (10), and (13),

$$A_a = \frac{\exp jkR}{i\lambda R} \; a_{1a} \int F \; ds \quad , \tag{16}$$

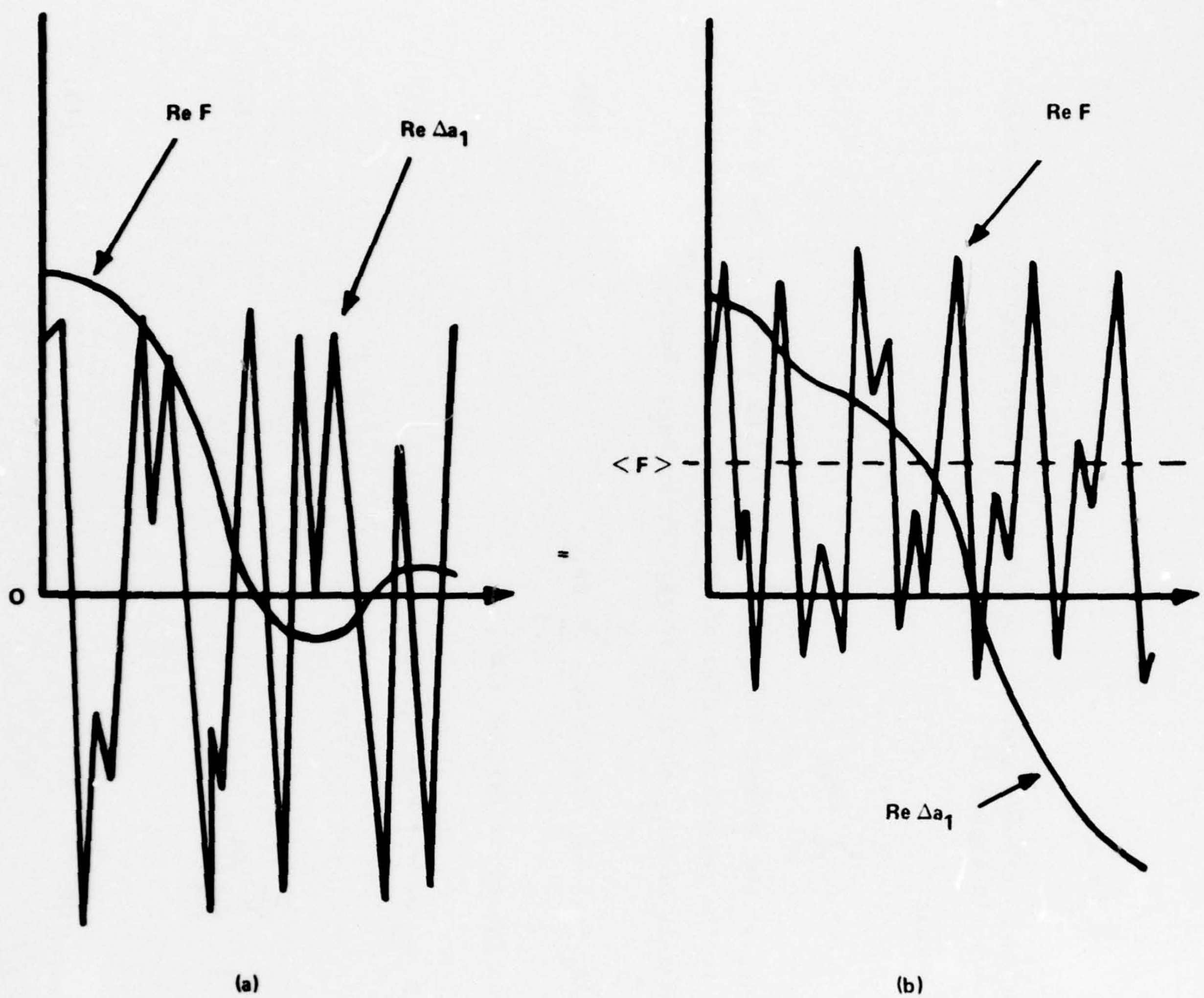

Figure 3. Equivalence of the integral in Equation (5) before (a) and after (b) reordering the integration differentials.

and

$$\Delta A = \frac{\exp jkR}{j\lambda R} \frac{1}{N} \delta\Delta a_1 \int F \, ds \quad . \tag{17}$$

Therefore, $I = g\, A^*A$ is given by

$$I(\alpha) = g \left(\frac{1}{\lambda R}\right)^2 \left| \int F \, ds \right|^2 \left| a_{1a} + \frac{1}{N} \delta\Delta a_1 \right|^2 \quad , \tag{18}$$

or

$$I(\alpha) = g \left(\frac{1}{\lambda R}\right)^2 \left| \int F \, ds \right|^2 \left[a_{1a}^2 + \frac{2}{N} a_{1a} \, \delta\mathrm{Re}\, \Delta a_1 + \frac{1}{N^2} |\delta\Delta a_1|^2 \right] \quad , \tag{19}$$

where the fact that $\delta\Delta a_1 + \delta\Delta a_1{}^* = 2\, \delta\mathrm{Re}\Delta a_1$. I is the far-field intensity which fluctuates as the target rotates. Thus, one may write

$$I(\alpha) = \langle I \rangle + \delta I_1(\alpha) + \delta I_2(\alpha) \quad , \tag{20}$$

where

$$\langle I \rangle = g \left(\frac{1}{\lambda R}\right)^2 \left| \int F \, ds \right|^2 \left[a_{1a}^2 + \frac{1}{N^2} \langle |\delta\Delta a_1|^2 \rangle \right] \quad , \tag{21}$$

$$\delta I_1 = g \left(\frac{1}{\lambda R}\right)^2 \left| \int F \, ds \right|^2 \frac{2}{N} a_{1a} \, \delta\mathrm{Re}\Delta a_1 \quad , \tag{22}$$

and

$$\delta I_2 = g \left(\frac{1}{\lambda R}\right)^2 \left| \int F \, ds \right|^2 \frac{1}{N^2} \left(|\delta\Delta a_1|^2 - \langle |\delta\Delta a_1|^2 \rangle \right) \quad . \tag{23}$$

Note that, since it fluctuates about zero, $\langle \delta\mathrm{Re}\Delta a_1 \rangle = 0$. The averaging relates to the variable α.

Now, turn to the derivation of $\delta\mathrm{Re}\Delta a_1$ and $|\delta\Delta a_1|^2$ in terms of surface properties. First, consider that the fluctuation Δa_1 in the n th cell is

$$\Delta a_{1n} = a_o \, p \exp j\phi_n - a_o \, p \langle \exp j\phi \rangle \quad .$$

Thus,

$$\delta\Delta a_1 = \sum_{n=1}^{N} \Delta a_{1n} = a_o\, p \sum_{n=1}^{N} (\exp j\phi_n - \langle \exp j\phi \rangle) \quad .$$

Now, choose the set $\{\phi_n\}$ such that $\delta\Delta a_1 \equiv \delta\Delta a_{10} = 0$, and reorder the summation to produce the set $\{\phi_i\}$ which runs from highest to lowest, but includes the same values as $\{\phi_n\}$:

$$\delta\Delta a_{10} = 0 = a_o\, p \sum_{i=1}^{N} (\exp j\phi_i - \langle \exp j\phi \rangle) \quad . \tag{24}$$

Now, let the surface orientation α change so that $\phi_i \to \phi_i + \delta\phi$ ($\delta\Delta a_1$ is no longer zero). Thus,

$$\delta\Delta a_1 = a_o\, p \sum_{i=1}^{N} \left[\exp j(\phi_i + \delta\phi_i) - \langle \exp j\phi \rangle\right] \quad .$$

Reordering the terms such that the summation runs from highest $\{\phi_i + \delta\phi_i\}$ to lowest (but keeping the same values) and calling this set $\{\phi_h\}$, $\delta\Delta a_1$ is expressed as

$$\delta\Delta a_1 = a_o\, p \sum_{h=1}^{N} (\exp j\phi_h - \langle \exp j\phi \rangle) \quad . \tag{25}$$

Due to the reordering, the absolute difference between ϕ_h and ϕ_i ($i = h$) is $|\delta\phi_h|$ which is much smaller, on the average, than $|\delta\phi_i|$. Therefore,

$$\delta\Delta a_1 = \delta\Delta a_1 - \delta\Delta a_{10} = a_o\, p \sum_{i=h=1}^{N} \left[\exp j(\phi_i + \delta\phi_h) - \langle \exp j\phi \rangle\right]$$

becomes

$$\delta\Delta a_1 \approx a_o\, p \sum_{i=h=1}^{N} \left[\exp j\phi_i\, (1 + j\delta\phi_h) - \langle \exp j\phi \rangle\right]$$

or

$$\delta\Delta a_1 \approx a_o\, p \sum_{i=h=1}^{N} (\exp j\phi_i)\, j\delta\phi_h \quad . \tag{26}$$

Use has been made of the fact that the sum over $\exp j\phi_i$ is $N \langle \exp j\phi \rangle$. Thus,

$$\mathrm{Im}\ \delta\Delta a_1 = a_o\ p \sum_{i=h=1}^{N} \cos\phi_i\ \delta\phi_h \quad ,$$

$$\mathrm{Im}\ \delta\Delta a_1 = a_o\ p \left[\sum_{\mathrm{pos}\ \phi_i} \cos\phi_i\ (\phi_h - \phi_i)_{\mathrm{pos}} \right.$$

$$\left. + \sum_{\mathrm{neg}\ \phi_i} \cos\phi_i\ (\phi_h - \phi_i)_{\mathrm{neg}} \right] \quad ,$$

$$\mathrm{Im}\ \delta\Delta a_1 = a_o\ p \langle \cos\phi \rangle \sum_{h=1}^{N/2} (\phi_{h\ \mathrm{pos}} + \phi_{n\ \mathrm{neg}}) \quad ,$$

where the subscripts pos and neg denote the value for ϕ_h for the h th cell (positive ϕ_i region) and the h + N/2 th cell (negative ϕ_i region), respectively. Defining

$$\delta_I\phi = \sum_{h=1}^{N/2} (\phi_{h\ \mathrm{pos}} + \phi_{h\ \mathrm{neg}}) \qquad (27)$$

enables one to write

$$\mathrm{Im}\ \delta\Delta a_1 = a_o\ p \langle \cos\phi \rangle\ \delta_I\phi \quad . \qquad (28)$$

Also, since $\Sigma\delta\phi_h = \Sigma\phi_h$,

$$\delta_I\phi = \sum_{j=1}^{N} \phi_j \quad ,$$

where $\{\phi_j\}$ is a random reordering of $\{\phi_h\}$. According to the formula for the fluctuation of a sum of N fluctuating terms, the rms value of $\delta_I\phi$ over all orientations α is

$$(\delta_I\phi)_{\mathrm{rms}} = N^{1/2}\ (\phi_j)_{\mathrm{rms}} \quad .$$

Due to equal probability of all $\phi_j \leq \phi_m$, the size-ordered array is linear, and it is simple to show that $(\phi_j)_{rms} = \phi_m/\sqrt{3}$, where ϕ_m is the maximum value of ϕ_j in the set of $N(\alpha)$ values which has been averaged over all orientations (all values of α). Thus,

$$(\delta_I\phi)_{rms} = N^{1/2}\ \phi_m/\sqrt{3} \quad . \tag{29}$$

Attention is now turned to Re $\delta\Delta a_1$. From Equation (26)

$$\mathrm{Re}\ \delta\Delta a_1 = -\ a_o\ p \sum_{i=h=1}^{N} \sin\phi_i\ \delta\phi_h \quad , \tag{30}$$

$$\mathrm{Re}\ \delta\Delta a_1 = -\ a_o\ p <\sin|\phi|> \left(\sum_{\mathrm{pos}\ \phi_i} \delta\phi_h - \sum_{\mathrm{neg}\ \phi_i} \delta\phi_h \right) \quad ,$$

$$\mathrm{Re}\ \delta\Delta a_1 = -\ a_o\ p <\sin|\phi|> \left[\sum_{\substack{\mathrm{pos}\ \phi_i \\ i=h}} (\phi_{h\ \mathrm{pos}} - \phi_i) - \sum_{\substack{\mathrm{neg}\ \phi_i \\ i=h}} (\phi_{h\ \mathrm{neg}} - \phi_i) \right] \quad , \tag{31}$$

or

$$\mathrm{Re}\ \delta\Delta a_1 = -\ a_o\ p <\sin|\phi|> \left[\sum_{h=1}^{N} (\phi_{h\ \mathrm{pos}} - \phi_{h\ \mathrm{neg}}) - N <|\phi_i|> \right] \quad . \tag{32}$$

Thus, defining $\delta_R\phi$ as the term in brackets,

$$\mathrm{Re}\ \delta\Delta a_1 = -\ a_o\ p <\sin|\phi|>\ \delta_R\phi \quad . \tag{33}$$

Now, define a set ϕ_i' such that $\phi_i' = \phi_h$ for $i = h$. Equation (32) becomes

$$\mathrm{Re}\ \delta\Delta a_1 = -a_o\ p <\sin|\phi|> \sum_{i=1}^{N} (|\phi_i'| - |\phi_i|) \quad .$$

Since

$$\sum_{i=1}^{N} |\phi_i| = N <|\phi|> \quad ,$$

then

$$\delta_R\phi = \sum_{i=1}^{N} |\phi_i'| - N<|\phi|> \quad .$$

By forming $<\delta_R^2>$ and noting that, for the rectangular probability function, $<|\phi|> = \sqrt{3}/2\ \phi_{rms}$, it is straightforward to show

$$(\delta_R\phi)_{rms} = N^{\frac{1}{2}}\ \phi_m/\sqrt{12} = \frac{1}{2}\ (\delta_I\phi)_{rms} \quad . \tag{34}$$

Whereas $\delta_I\phi$ is a measure of whether a particular set $\{\phi_h\}$ inclines toward positive or negative values, $\delta_R\phi$ is a measure of whether the total range of $\{\phi_h\}$ is greater or less than that of $\{\phi_i\}$. A study of Equations (27) and (32) makes this clear. Thus, $\delta_I\phi$ and $\delta_R\phi$ both fluctuate about zero and have **similar** rms value, but they are independent (random with respect to one another).

Therefore, $|\delta\Delta a_1|^2$ is given by

$$|\delta\Delta a_1|^2 = a_o^2\ p^2 \left[<\cos\phi>^2\ (\delta_I\phi)^2 + <\sin|\phi|>^2\ (\delta_R\phi)^2\right] \quad , \tag{35}$$

and, hence, defining

$$B = g \left(\frac{1}{\lambda R}\right)^2 \left|\int F\ ds\right|^2 \quad , \tag{36}$$

and noting that $a_{1a} = \langle a_1 \rangle = \langle \cos\phi \rangle\, a_o\, p$, Equations (22), (23), (33), (35), and (36) give

$$(\delta I)_1 = -B\ 2\ a_o^2\ p^2\ \frac{1}{N} \langle \cos\phi \rangle \langle \sin|\phi| \rangle\ \delta_R\phi \tag{37}$$

and

$$(\delta I)_2 = B\ a_o^2\ p^2 \left(\frac{1}{N}\right)^2 \left\{ \langle \cos\phi \rangle^2 \left[(\delta_I\phi)^2 - \langle (\delta_I\phi)^2 \rangle \right] + \langle \sin|\phi| \rangle^2 \left[(\delta_R\phi)^2 - \langle (\delta_R\phi)^2 \rangle \right] \right\} \quad . \tag{38}$$

Because $\delta I = (\delta I)_1 + (\delta I)_2$,

$$(\delta I)_{rms} = \left[\langle (\delta I)_1^2 + 2\ (\delta I)_1 (\delta I)_2 + (\delta I)_2^2 \rangle \right]^{1/2} \quad . \tag{39}$$

The product $(\delta I)_1 (\delta I)_2$ vanishes upon taking the average since $\delta_R\phi$ fluctuates about zero and $(\delta I)_2$ is a linear combination of constants, and $(\delta_R\phi)^2$, and $(\delta_I\phi)^2$ which is independent of $\delta_R\phi$. Thus,

$$(\delta I)_{rms} = \left[\langle (\delta I)_1^2 \rangle + \langle (\delta I)_2^2 \rangle \right]^{1/2} \quad . \tag{40}$$

To use Equation (40), the expression $\langle [(\delta\phi)^2 - \langle (\delta\phi)^2 \rangle]^2 \rangle$, where δ may be subscripted with I or R, must be reduced to one with known parameters. Note that

$$\langle \left[(\delta\phi)^2 - \langle (\delta\phi)^2 \rangle \right]^2 \rangle = \langle (\delta\phi)^4 - 2\ (\delta\phi)^2 \langle (\delta\phi)^2 \rangle + \langle (\delta\phi)^2 \rangle^2 \rangle$$

$$= \langle (\delta\phi)^4 \rangle - \langle (\delta\phi)^2 \rangle^2 \quad .$$

Appendix B shows that for $\delta\phi = \delta_I\phi$, this further reduces to

$$\langle \left[(\delta\phi)^2 - \langle (\delta\phi)^2 \rangle \right]^2 \rangle = (N^2 - N)\ \langle \phi_i^2 \rangle^2 \quad . \tag{41}$$

Since $\langle\phi_i^2\rangle^2 = (\phi_i)^4_{rms} = (\phi_m/\sqrt{3})^4 = \phi_m^4/9$ for $\delta_I\phi$, and $(\delta_R\phi)_{rms} = 1/2\ (\delta_I\phi)_{rms}$ ($\phi_i \rightarrow |\phi_i'| - |\phi_i|$ for $\delta\phi \rightarrow \delta_R\phi$), then

$$\langle(\delta_I\phi)^4\rangle - \langle(\delta_I\phi)^2\rangle^2 = 16\langle(\delta_R\phi)^4\rangle - \langle(\delta_R\phi)^2\rangle^2 = (N^2 - N)\ \phi_m^4/9 \quad . \tag{42}$$

Hence, using Equations (34), (37), (38), and (42),

$$\langle(\delta I)_1^2\rangle = 4\ B^2\ a_o^4\ p^4 \left(\frac{1}{N}\right)^2 \langle\cos\phi\rangle^2\langle\sin|\phi|\rangle^2\ N\ \phi_m^2/12 \tag{43}$$

and

$$\langle(\delta I)_2^2\rangle = B^2\ a_o^4\ p^4 \left(\frac{1}{N}\right)^4 \left[(\langle\cos\phi\rangle^4 + \frac{1}{16}\langle\sin|\phi|\rangle^4)\ (N^2 - N)\ \phi_m^4/9\right] \quad . \tag{44}$$

The cross-term $2/16\ \langle\cos\phi\rangle^2\langle\sin|\phi|\rangle^2\ [(\delta_I\phi)^2 - \langle(\delta_I\phi)^2\rangle][(\delta_R\phi)^2 - \langle(\delta_R\phi)^2\rangle]$ has been omitted because each factor fluctuates about zero and is independent; hence, it vanishes upon averaging.

Noting that $a_{1a}^2 = a_o^2\ p^2\ \langle\cos\phi\rangle^2$ and using Equations (21), (34), (35), and (36), $\langle I\rangle$ is given by

$$\langle I\rangle = B\left[a_o^2\ p^2\ \langle\cos\phi\rangle^2 + \left(\frac{1}{N}\right)^2 a_o^2\ p^2\ (\langle\cos\phi\rangle^2 + \frac{1}{4}\langle\sin|\phi|\rangle^2)\ N\ \phi_m^2/3\right]$$

or

$$\langle I\rangle = B\ a_o^2\ p^2 \left[\langle\cos\phi\rangle^2 + \frac{1}{N}(\langle\cos\phi\rangle^2 + \frac{1}{4}\langle\sin|\phi|\rangle^2)\ \phi_m^2/3\right] \quad . \tag{45}$$

Using Equations (40), (43), and (44), $(\delta I)_{rms}$ is given by

$$(\delta I)_{rms} = B\ a_o^2\ p^2\left(\frac{1}{N}\right)N^{1/2}\ \phi_m/\sqrt{3}\ \Bigg\{\langle\cos\phi\rangle^2\langle\sin|\phi|\rangle^2 + \left(\frac{1}{N}\right)^2 (\langle\cos\phi\rangle^4 + \frac{1}{16}\langle\sin|\phi|\rangle^4)(\phi_m^2/3)\ (N - 1)\Bigg\}^{1/2}$$

or

$$(\delta I)_{rms} = B\, a_o^2\, p^2 \frac{\phi_m}{(3N)^{1/2}} \left\{ <\cos\phi>^2 <\sin|\phi|>^2 + \left(<\cos\phi>^4 + \frac{1}{16} <\sin|\phi|>^4\right) (\phi_m^2/3) \left(\frac{1}{N} - \frac{1}{N^2}\right) \right\}^{1/2} . \quad (46)$$

Now,

$$<\cos\phi> = \frac{1}{\phi_m} \int_0^{\phi_m} \cos\phi \; d\phi = \frac{\sin\phi_m}{\phi_m} \quad (47)$$

and

$$<\sin|\phi|> = \frac{1}{\phi_m} \int_0^{\phi_m} \sin\phi \; d\phi = \frac{1 - \cos\phi_m}{\phi_m} \quad . \quad (48)$$

The spread of ϕ_m is $\pm\ \phi_m/\sqrt{N}$, thus, for $N \gtrsim 100$, this spread is negligible.

Thus, from Equations (45) and (46),

$$\boxed{\frac{(\delta I)_{rms}}{<I>} = \frac{\frac{\phi_m}{(3N)^{1/2}} \left[<\cos\phi>^2 <\sin|\phi|>^2 + \left(<\cos\phi>^4 + \frac{1}{16} <\sin|\phi|>^4\right) (\phi_m^2/3) \left(\frac{1}{N} - \frac{1}{N^2}\right) \right]^{1/2}}{<\cos\phi>^2 + \frac{1}{N} \left(<\cos\phi>^2 + \frac{1}{4} <\sin|\phi|>^2\right) \phi_m^2/3}} \quad . \quad (49)$$

Thus, the ratio of far-field intensity fluctuation to the average is uniquely determined by the rms phase fluctuation $= \phi_m/\sqrt{3}$ and the number of decorrelation cells $N = S/\Delta s_c$. Appendix C presents a derivation of N in terms of surface properties.

Case Where ϕ is Due to Surface Roughness: A scheme will now be developed to deduce the rms height variation and average absolute slope of a surface (where $\phi_m = -\ 2\ k\ h_m$) by measurement of $(\delta I)_{rms}/<I>$ at different wavelengths. First, imagine a sweeping through a broad wavelength band. Whenever $\phi_m = 4\ \pi\ h_m/\lambda \rightarrow \pi$, then $<\cos\phi> \rightarrow 0$ and Equation (49) becomes

$$\frac{(\delta I)_{rms}}{<I>} \rightarrow \frac{\frac{1}{N^{1/2}} \frac{\phi_m^2}{3} \left[<\sin|\phi|>^4 \left(\frac{1}{N} - \frac{1}{N^2}\right)\right]^{1/2}}{\frac{1}{N} <\sin|\phi|>^2 \frac{\phi_m^2}{3}} \quad ,$$

which, for large N, is

$$\boxed{\frac{(\delta I)_{rms}}{\langle I\rangle} \rightarrow 1 \ , \ \lambda \rightarrow \lambda_1 = 4\ h_m = 4\ \sqrt{3}\ h_{rms}} \quad . \tag{50}$$

Thus, $\lambda_1 = 4\sqrt{3}\ h_{rms}$ is the condition on λ for which $(\delta I)_{rms}/\langle I\rangle$ first reaches unity as λ is decreased from larger values. This condition uniquely determines h_{rms}.

Now, imagine λ is increased well beyond $2\ \lambda_1$. In this region, $\phi_m \ll \pi/2$ (or $h_m \ll \lambda_3/8$); hence, ℓ_c is given by Equation (C-6) of Appendix C. Thus,

$$N = S/\Delta s_c = S/\ell_c^2 \rightarrow S \left\langle \left|\frac{dh}{d\ell}\right| \right\rangle^2 / h_m^2 \quad . \tag{51}$$

Also, due to smallness of ϕ_m and 1/N, Equation (49) becomes

$$\frac{(\delta I)_{rms}}{\langle I\rangle} \approx \frac{\frac{\phi_m}{(3N)^{1/2}} \left[\langle\cos\phi\rangle^2 \langle\sin|\phi|\rangle^2\right]^{1/2}}{\langle\cos\phi\rangle^2}$$

or

$$\frac{(\delta I)_{rms}}{\langle I\rangle} \approx \frac{\phi_m}{(3N)^{1/2}} \frac{\langle\sin|\phi|\rangle}{\langle\cos\phi\rangle}$$

or, since ϕ is small and $\phi_m = 4\ \pi\ h_m/\lambda_3$,

$$\frac{(\delta I)_{rms}}{\langle I\rangle} \approx \frac{4\ \pi\ h_m}{\lambda_3\ (3N)^{1/2}} \left(\frac{\frac{4\ \pi\ h_m}{2\ \lambda_3}}{1}\right) .$$

Further, using $h_m = \sqrt{3}\ h_{rms}$ and Equation (51),

$$\boxed{\frac{(\delta I)_{rms}}{\langle I\rangle} \approx \frac{3(4\pi)^2}{2S^{1/2}} \left(\frac{h_{rms}^3}{\lambda_3^2}\right) \Big/ \left\langle\left|\frac{dh}{d\ell}\right|\right\rangle \ , \ \lambda_3 \gg 8\sqrt{3}\ h_{rms}} \quad . \tag{52}$$

An intermediate value of λ, e.g., λ_2, is chosen such that $\phi_m = \pi/2$. At this point, $\lambda_2 = 8\ h_m = 8\sqrt{3}\ h_{rms}$ and Equation (C-6) again yields Equation (51). Also,

$$\langle \cos\phi \rangle = \langle \sin|\phi| \rangle = 2/\pi \quad .$$

Hence, for large N, Equation (49) becomes

$$\frac{(\delta I)_{rms}}{\langle I \rangle} \approx \frac{\dfrac{\pi}{2(3N)^{1/2}} \left[\left(\dfrac{\pi}{2}\right)^4 + \left(\dfrac{\pi}{2}\right)^4 \dfrac{\pi^2}{(4)(3)} \left(\dfrac{1}{N}\right)\right]^{1/2}}{\left(\dfrac{\pi}{2}\right)^2 + \dfrac{1}{N}\left(\dfrac{\pi}{2}\right)^2 \dfrac{\pi^2}{(4)(3)}}$$

or

$$\boxed{\frac{(\delta I)_{rms}}{\langle I \rangle} \approx \frac{\pi/2}{(3N)^{1/2}} = \frac{\pi\ h_{rms}}{2S^{1/2}} \Big/ \langle |\frac{dh}{d\ell}| \rangle \quad , \ \lambda_2 = 8\sqrt{3}\ h_{rms}} \quad . \qquad (53)$$

Any two of Equations (50), (52), or (53) can be used to solve for h_{rms} and $\langle |dh/d\ell| \rangle$:

$$\boxed{h_{rms} = \frac{\lambda_1}{4\sqrt{3}} = \frac{\lambda_2}{8\sqrt{3}}} \quad , \qquad (54)$$

and

$$\boxed{\langle |\frac{dh}{d\ell}| \rangle = \frac{\pi\ \lambda_2}{S^{1/2}\ 16\sqrt{3}} \Big/ \left(\frac{(\delta I)_{rms}}{\langle I \rangle}\right)_{at\ \lambda_2}} \quad ; \qquad (55)$$

also,

$$\boxed{\langle |\frac{dh}{d\ell}| \rangle = \frac{\pi^2}{64\sqrt{3}\ S^{1/2}} \frac{\lambda_2^3}{\lambda_3^2} \Big/ \left(\frac{(\delta I)_{rms}}{\langle I \rangle}\right)_{at\ \lambda_3}} \quad , \qquad (56)$$

where $\lambda_1 = \lambda_2/2$ is the largest value of λ for which $(\delta I)_{rms}/\langle I \rangle = 1$, and λ_3 is any value of λ which is much greater than λ_2.

A sketch of the behavior of $(\delta I)_{rms}/\langle I \rangle$ with λ is shown in Figure 4. Calculations used to aid in this illustration are presented in Appendix D.

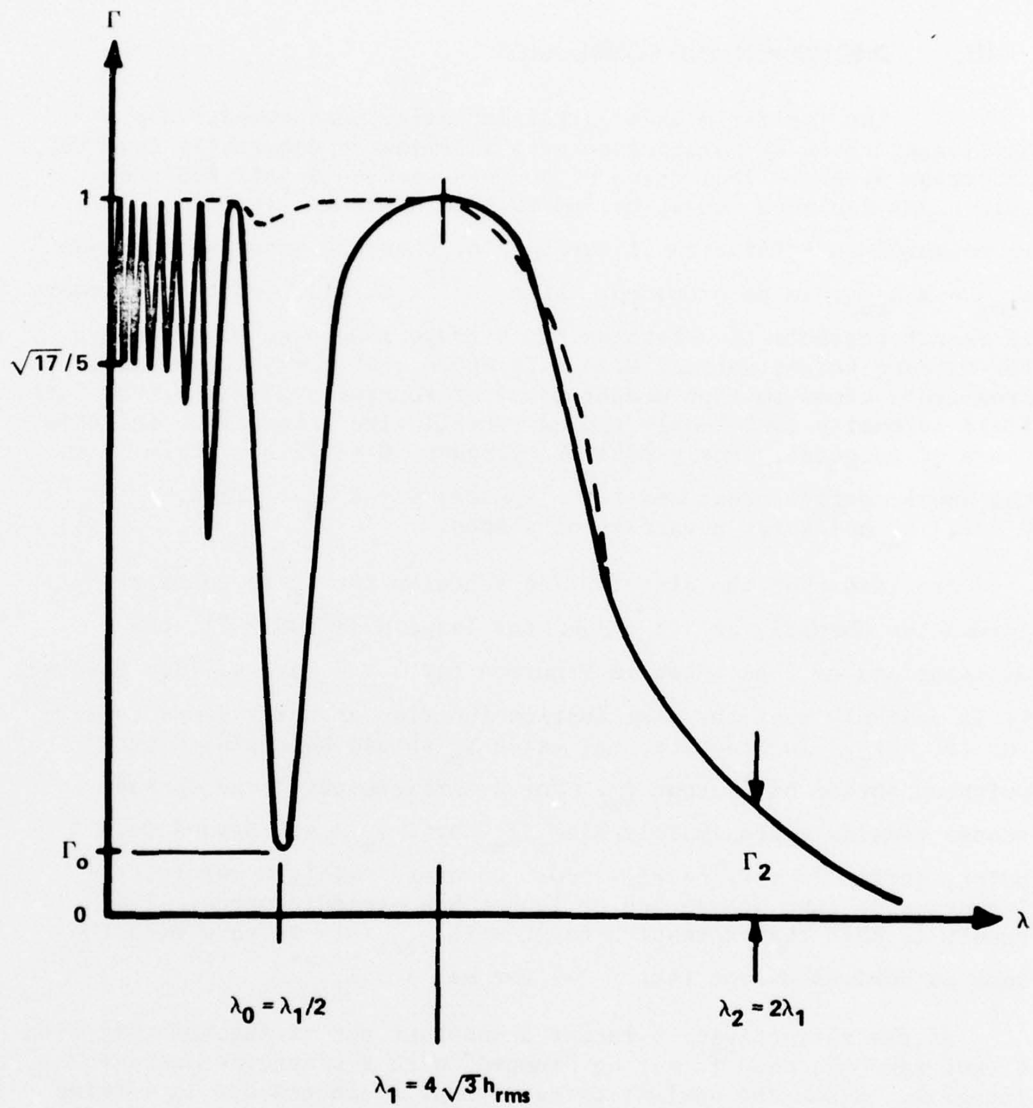

Figure 4. Behavior of $\Gamma = (\delta I)_{rms}/\langle I\rangle$ versus λ. NOTE: The perforated line is the behavior of Γ when Φ_m is replaced by a spread so that the probability density function for Φ_i does not have a sharp cutoff at Φ_m. In the figure,

$$\Gamma_2 = \pi\, h_{rms}/(2S^{1/2}\ \langle|dh/d\ell|\rangle)$$

and

$$\Gamma_o = \pi^2\, h_{rms}^2 \left[1 + \ln(8\sqrt{3}\, h_{rms}/\lambda_o)\right]^2/(4S\ \langle|dh/d\ell|\rangle^2)\ .$$

N is assumed to be very large.

III. DISCUSSION AND CONCLUSION

The far-field backscatter intensity measurements for a particular class of rotating targets illuminated coherently from the detection site are indicative of certain surface detail features. This class includes cones, cylinders, and spheres. If $(\delta I)_{rms}/\langle I\rangle$ is measured as a function of wavelength, then the surface roughness $\phi_{rms} = k\, h_{rms}$ can be obtained. Also, $S^{1/2}\langle|dh/d\ell|\rangle$ can be obtained. It is not possible to determine the average roughness slope unless the exposed target surface area S is known. This may be estimated from radar cross-section measurements or Fourier analysis of the far-field intensity (which only yields overall size information for this class of targets). For a cone or cylinder, $S = \pi/2\ S_a$, where S_a is the cross-section area; and for a sphere, $S = 2\ S_a$. Thus, $S \simeq 1.7\ S_a$ holds for a variety of shapes.

Provided that the distribution function for ϕ_i is constant but terminates abruptly at $\pm\ \phi_m$, then, for large N ($\phi_m^2/3N \gg 1$), the oscillations of Γ as shown in Figure 4 for $\lambda \leq \lambda_o$ are valid. However, it is unlikely that the distribution function abruptly drops to zero for $|\phi| > \phi_m$. In practice, the value ϕ_m should be replaced by a weighted spread of ϕ about ϕ_m. For a sufficiently large spread, $\langle\cos\phi\rangle$ remains approximately $\sin\phi_m/\phi_m$ until $\phi_m \to \pi$. Beyond this point, $\langle\cos\phi\rangle$ is very nearly zero. However, $\langle\sin|\phi|\rangle$ maintains a larger value than $\langle\cos\phi\rangle$ and no longer has periodic zeros. The result is that the expression for $\Gamma = (\delta I)_{rms}/\langle I\rangle$ is very much the same as derived except that $\Gamma \simeq 1$ for all $\lambda < \lambda_1$.

If the reflectivity p is not a constant but varies smoothly with θ (not random), then it may be "lumped" with F under the surface integral. Since the surface integral over F cancels out in forming the ratio, it is not important in this analysis.

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Appendix A. JUSTIFICATION THAT $\phi \approx -2k\langle h\rangle$ OVER A DECORRELATION AREA CELL WHEN $\theta > 0$ AND PHASE IS DETERMINED BY SURFACE ROUGHNESS

Consider a constructed surface [perforated line in Figure A-1(a)] which represents the average surface observed at an angle θ. Examine one decorrelation cell for $\lambda > 8\,h_m$ (where the surface departs from $h = 0$ and then returns). Reflection from the real surface at distance r_r appears to come from the constructed surface at distance r. The path difference between the wave from the real surface and the wave if it struck the constructed surface is

$$\Delta p = r - r_r - H \quad .$$

Now,

$$H = \frac{-h}{\sin\left(\theta - \psi_i\right)} = \frac{-h}{\sin\left(\theta - 2\theta_i - \frac{\pi}{2}\right)} \quad ,$$

and

$$r - r_r = H \sin \psi_i = \frac{h}{\sin\left(\theta - 2\theta_i - \frac{\pi}{2}\right)} \sin\left(2\theta_i - \frac{\pi}{2}\right) \quad .$$

Thus,

$$\Delta p = - \frac{h}{\sin\left(\theta - 2\theta_i - \frac{\pi}{2}\right)} \left[\sin\left(2\theta_i - \frac{\pi}{2}\right) - 1\right]$$

or, using trigonometric identities,

$$\Delta p = -2\,h\,\frac{\cos^2\theta_i}{\cos(2\theta_i - \theta)} \quad .$$

This does not depart significantly from $-2\,h$ except for $\theta \geq \pi/4$, in which case (for small roughness slope relative to θ so that $\theta_i \approx \theta$),

$$\langle\Delta p\rangle \approx -2\,\langle h\rangle_i \cos\theta_i \quad . \qquad \text{(A-1)}$$

The Rayleigh-Sommerfield Diffraction formulation yields

$$A = \frac{1}{j\lambda} \int_S a \, [\exp(jkr)/r] \cos\theta \, ds \quad , \tag{A-2}$$

where the surface field a is given by

$$a = a_o \, p \exp[jkr + j\phi(r)] \quad , \tag{A-3}$$

where a_o is a real constant representing the incident field, p is the reflectance (includes a factor $e^{j\pi} = -1$ if the surface is metallic and inverts the phase by 180 deg), and ϕ is the phase of a at the constructed surface due to roughness. Hence, dividing the surface into N decorrelation cells and using Equations (A-1) and (A-3),

$$A = \frac{1}{j\lambda} \sum_{i=1}^{N} \Delta s_i \, [\exp(jk\langle r\rangle_i)/r] a_o \, p \exp[jk\langle r\rangle_i - j2k\langle h\rangle_i \cos\theta_i] \cos\theta_i \; .$$

Examination of Figure A-1(b) will show that the intensity of the reflected light at the constructed surface peaks above the projection of the cell's midpoint on the constructed surface. Thus, $\langle r\rangle_i$ is a weighted average and is equal to $r_i + \Delta r_i$, where r_i is r at the projected midpoint on the surface and Δr_i is a small value. If h is negative, Δr_i is positive; and if h is positive, the intensity peaks below the projected midpoint and Δr_i is negative. The magnitude of Δr_i increases as h increases and/or $\cos\theta$ decreases. Thus, A becomes

$$A \simeq \frac{a_o \, p}{j \, \lambda} \frac{\exp jkR}{R} \sum_{i=1}^{N} \Delta s_i \exp[j2(kr_i - R)]$$

$$\times \exp j(-2k\langle h\rangle_i \cos\theta_i + 2k \, \Delta r_i) \cos\theta_i \quad . \tag{A-4}$$

The effective cell phase ϕ_i is defined as

$$\phi_i = -\,2\,k \left[\langle h\rangle_i \cos\theta_i + \Delta r_i(\theta_i)\right] \quad . \tag{A-5}$$

Due to the compensating nature of Δr_i, ϕ_i does not fall off with $\cos\theta$ as fast as $2k \langle h \rangle \cos\theta$. The assumption is made that for $\cos\theta_i$ sufficiently large that the contribution to the integral is significant, then

$$\phi_i \approx - 2k \langle h \rangle_i \quad . \tag{A-6}$$

Thus,

$$A \approx \frac{a_o p}{j\lambda R} \exp jkR \sum_{i=1}^{N} \Delta s_i \; F(r_i,\theta_i) \exp j\phi_i \quad , \tag{A-7}$$

where

$$F(r_i,\theta_i) = \exp[j2(kr_i - R)] \cos\theta_i \tag{A-8}$$

is defined as the *surface curvature factor.*

It is also assumed that the cell slope deviation from θ is sufficiently small that shading due to height fluctuations is not significant for $\cos\theta$ sufficiently large that contribution to the surface integral is important.

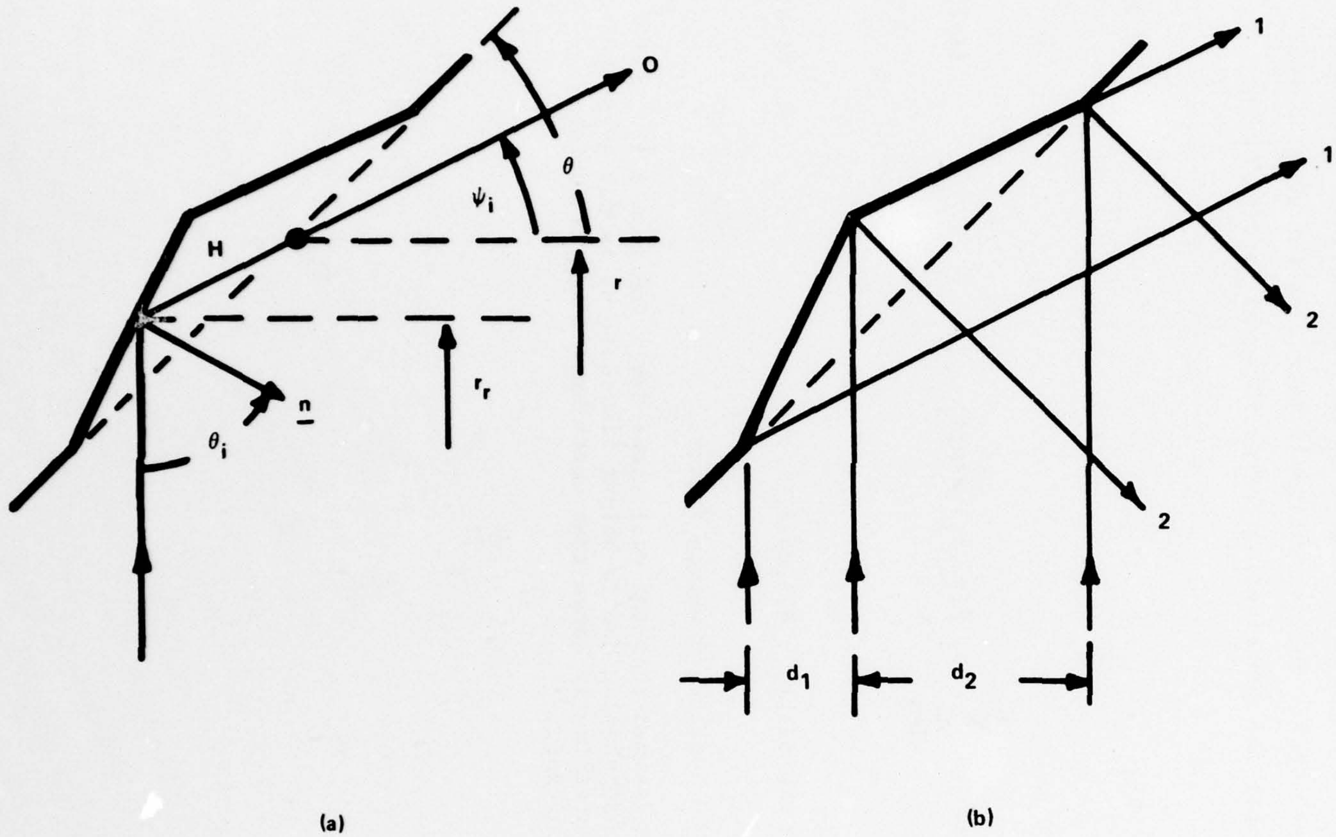


Figure A-1. A plane wave is assumed to strike the surface of a decorrelation cell from below. (An observer at 0 detects the phase change.)

Appendix B. EVALUATION OF $<(\delta\phi)^4> - <(\delta\phi)^2>^2$

Let ϕ_i be the phase of site i. Also take $\delta\phi$ as representing $\delta_I\phi$.

$$\delta\phi = \sum_{i=1}^{N} \phi_i \quad . \tag{B-1}$$

Thus,

$$(\delta\phi)^2 = \sum_{i=1}^{N} \phi_i^2 + \sum_{i=1}^{N} \sum_{j\neq 1} \phi_i \phi_j \tag{B-2}$$

or

$$(\delta\phi)^2 = N <\phi_i^2> + \sum_{i=1}^{N} \sum_{j\neq 1} \phi_i \phi_j \quad , \tag{B-3}$$

so that

$$<(\delta\phi)^2> = N <\phi_i^2> \quad . \tag{B-4}$$

Also,

$$(\delta\phi)^4 = N^2 <\phi_i^2>^2 + 2N <\phi_i^2> \sum_{i=1}^{N} \sum_{j\neq 1} \phi_i \phi_j + \left(\sum_{i=1}^{N} \phi_i\right)^2 \left(\sum_{j\neq 1} \phi_j\right)^2 . \tag{B-5}$$

Thus,

$$<(\delta\phi)^4> = N^2 <\phi_i^2>^2 + 0 + <\left(\sum_{i=1}^{N} \phi_i\right)^2> <\left(\sum_{j\neq 1} \phi_j\right)^2> \quad ,$$

where the factors of the last term average separately because ϕ_i and ϕ_j, ($j\neq i$) are independent. Thus, using Equations (B-1) and (B-4), the last equation becomes

$$<(\delta\phi)^4> = N^2 <\phi_i^2>^2 + N\ (N - 1) <\phi_i^2>^2$$

or

$$<(\delta\phi)^4> = (2\ N^2 - N) <\phi_i^2>^2 \quad . \tag{B-6}$$

Therefore,

$$<(\delta\phi)^4> - <(\delta\phi)^2>^2 = (N^2 - N) <\phi_i^2>^2 \quad . \tag{B-7}$$

Appendix C. AN EXPRESSION FOR $N = S/\Delta s_c$

The quantity ϕ_m is the average of the maximum value of ϕ_i in all decorrelation area cells $\Delta s_i \equiv \ell_{ci}^2$. Let ϕ be piecewise linear so that the surface field Re a_1 varies sinusoidally over a cell as shown in Figure C-1. The amplitude and periodicity is random from cell to cell. For small ϕ_{mi}, the periodicity of Re a_1 is the same as that of ϕ_{mi}. For large ϕ_{mi}, Re a_1 may oscillate through many cycles as ϕ_i changes and, hence, ℓ_{ci} is small.

Since ϕ_i fluctuates randomly about zero from $-\phi_m$ to ϕ_m, then ϕ_{mi} fluctuates from 0 to $2\phi_m$.

$$\phi_{mi} = (1 + \delta_i')\,\phi_m \quad , \; -1 < \delta_i' < 1 \quad , \tag{C-1}$$

where δ_i' is a bounded, random fluctuation from cell to cell, and the average cell length is

$$\ell_c = \frac{1}{2}\int_{-1}^{1} \ell_{ci}\, d\delta' \quad . \tag{C-2}$$

Defining ℓ_{pvi} as the surface distance for which ϕ rises linearly from 0 to ϕ_{im}, then one can see that ℓ_{pvi} is the distance from "peak" Re Δa_1 to the "valley" of Re Δa_1 in the region of the i th cell (for small ϕ_m). Therefore,

$$\text{Small } \phi_m \text{ condition: } \; 0 < \phi_{mi} = (1 + \delta_i')\phi_m < \pi \Rightarrow \; \ell_{ci} = \ell_{pvi} \quad ;$$

$$\text{Large } \phi_{mi} \text{ condition: } \; \pi < \phi_{mi} = (1 + \delta_i')\phi m < \infty \Rightarrow \; \ell_{ci} = \pi / \left|\frac{d\phi}{d\ell}\right|_i \quad .$$

Restating these conditions along with that of Equation (C-1) gives

i) $\phi_m < \pi/2$ condition: $\ell_{ci} = \ell_{pvi}$;

ii) $\phi_m > \pi/2$, small δ_i' condition: $-1 < \delta_i' < \frac{\pi}{\phi_m} - 1 \Rightarrow \ell_{ci} = \ell_{pvi}$;

iii) $\phi_m > \pi/2$, large δ'_i condition: $\frac{\pi}{\phi_m} - 1 < \delta'_i < 1 \Rightarrow \ell_{ci}$

$$= \pi \Big/ \left|\frac{d\phi}{d\ell}\right|_i \quad .$$

Thus, for $\phi_m < \pi/2$, condition i) implies

$$\ell_c = \frac{1}{2} \int_{-1}^{1} \ell_{pv}(\delta')\, d\delta' = \ell_{pv} \ , \ \phi_m < \pi/2 \quad , \tag{C-3}$$

where ℓ_{pv} is the average value of ℓ_{pvi}. For $\phi_m > \pi/2$, and noting that $|d\phi/d\ell|_i = \phi_{mi}/\ell_{pvi}$, conditions ii) and iii) imply

$$\ell_c = \frac{1}{2} \left[\int_{-1}^{\pi/\phi_m - 1} \ell_{pv}(\delta')\, d\delta' + \pi \int_{\pi/\phi_m - 1}^{1} \ell_{pv}(\delta') \frac{d\delta'}{\phi_m(\delta')} \right] \quad ,$$

$$\ell_c = \frac{1}{2} \left[\ell_{pv} \frac{\pi}{\phi_m} + \pi \frac{\ell_{pv}}{\phi_m} \int_{\pi/\phi_m - 1}^{1} \frac{d\delta'}{1 + \delta'} \right] \quad , \tag{C-4}$$

or

$$\ell_c = \frac{\pi\, \ell_{pv}}{2\, \phi_m} \left[1 + \ln \frac{2}{\pi/\phi_m} \right] \ , \quad \phi_m > \pi/2 \quad . \tag{C-5}$$

Since $N = S/\Delta s_c = S/\ell_c^2$, Equations (C-3) and (C-5) determine N in terms of ϕ_m and $\ell_{pv} = \phi_m/\langle|d\phi/d\ell|\rangle$. A sketch of the behavior of ℓ_c with ϕ_m is shown in Figure C-2. Only when ϕ_m is small is a decorrelation length based on the autocorrelation of ϕ the same as that based on the autocorrelation of Δa_1.

Consider the case where the variation of ϕ is due to surface roughness. For this case, $\phi_m = 2k\, h_m = 4\pi\, h_m/\lambda$. Equation (C-3) becomes

$$\ell_c = \ell_{pv} = \phi_m \Big/ \langle\left|\frac{d\phi}{d\ell}\right|\rangle$$

or

$$\ell_c = h_m \Big/ \left\langle \left| \frac{dh}{d\ell} \right| \right\rangle \quad , \; h_m < \lambda/8 \quad . \tag{C-6}$$

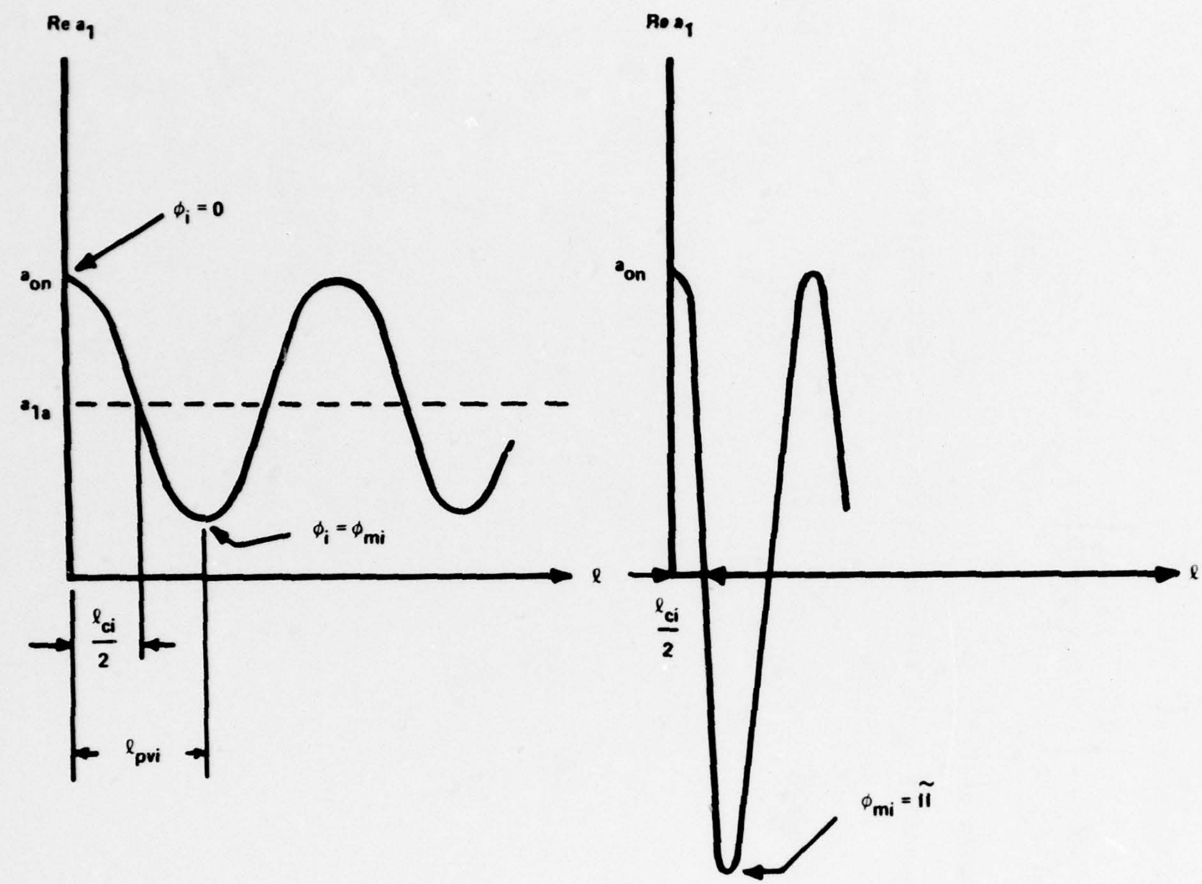
Equation (C-5) becomes

$$\ell_c = \frac{\lambda}{8 \left\langle \left| \frac{dh}{d\ell} \right| \right\rangle} \left[1 + \ell n \left(\frac{8\, h_m}{\lambda} \right) \right], \; h_m > \lambda/8 \quad , \tag{C-7}$$

and N is given by S/ℓ_c^2.

The quantity h_m is related to the rms height variation (from zero) by

$$h_{rms} = h_m/\sqrt{3} \quad . \tag{C-8}$$



(a) SMALL ϕ_{mi} (b) LARGE ϕ_{mi}

Figure C-1. Field variation along surface distance ℓ and its relation to the decorrelation cell length ℓ_{ci}.

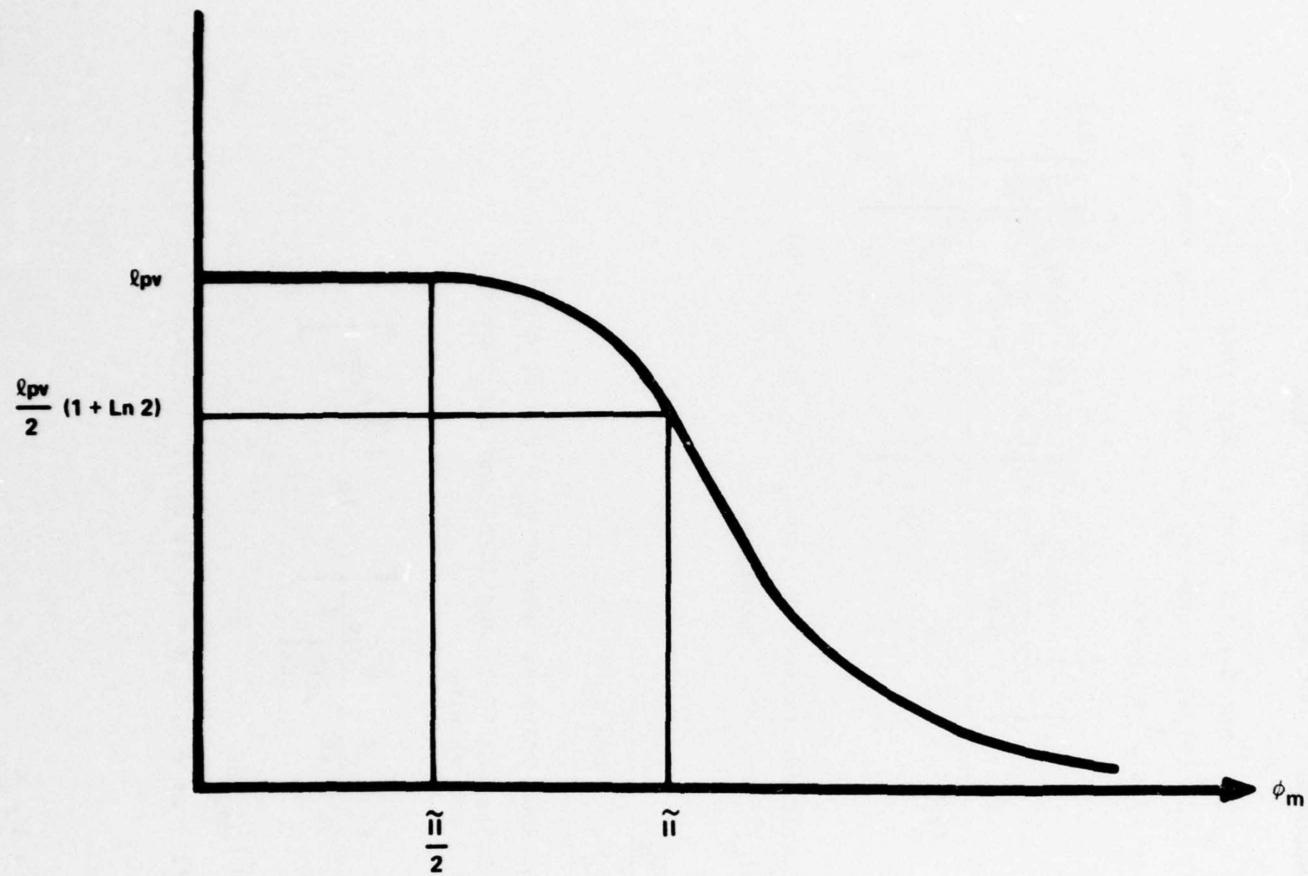


Figure C-2. Variation of ℓ_c with ϕ_m . (The decorrelation length ℓ_c is not proportional to the autocorrelation length of ϕ except for $\phi_m < \pi/2$.)

Appendix D. THE BEHAVIOR OF $(\delta I)_{rms}/\langle I\rangle$ WITH λ

Consider $\phi_m > \pi$, but N sufficiently large that $\phi_m/N \ll 1$. Using Equations (47) and (48) with Equation (49) and dividing numerator and denominator by $\sin^2 \phi_m$ gives

$$\frac{(\delta I)_{rms}}{\langle I\rangle} = \frac{\phi_m}{(3N)^{1/2}} \frac{\left\{ \frac{(1-\cos\phi_m)^2}{\sin^2 \phi_m} + \left[1 + \frac{(1-\cos\phi_m)^4}{16 \sin^4 \phi_m}\right]\frac{\phi_m^2}{3N} \right\}^{1/2}}{1 + \left[1 + \frac{(1-\cos\phi_m)^2}{4 \sin^2 \phi_m}\right]\frac{\phi_m^2}{3N}} .$$

From trigonometric identities, $1 - \cos\phi_m = 2 \sin^2\phi_m/2$ and $\sin\phi_m = 2 \sin\phi_m/2 \cos\phi_m/2$. Therefore,

$$\frac{(1-\cos\phi_m)^2}{\sin^2\phi_m} = \frac{\sin^2\phi_m/2}{\cos^2\phi_m/2} ,$$

which has a minimum value of zero at $\phi_m = 2\, n\, \pi$, $n = 1, 2, 3, \ldots$). Hence, using Equations (C-7) and (C-8) and the fact that $\phi_m = 4\, \pi\, h_m/\lambda$, one obtains

$$\frac{(\delta I)_{rms}}{\langle I\rangle} \simeq \frac{\phi_m^2}{3N} = \frac{\pi^2}{4S} \frac{h_{rms}^2}{\left\langle \left|\frac{dh}{d\ell}\right|^2 \right\rangle} \left[1 + \ln \frac{8\sqrt{3}\, h_{rms}}{\lambda}\right]^2 , \qquad \text{(D-1)}$$

where $\lambda = 2\sqrt{3}\, h_{rms}/n$.

When $\phi_m/2 \to n\pi + \pi/2$, $n = 0, 1, 2, 3, \ldots$, then $\sin^2(\phi_m/2)/\cos^2(\phi_m/2) \to \infty$, and

$$\frac{(\delta I)_{rms}}{\langle I\rangle} \to 1 . \qquad \text{(D-2)}$$

Hence, for $\phi_m > \pi$ but $\phi_m/N \ll 1$, $(\delta I)_{rms}/\langle I\rangle$ oscillates between the limits given by Equations (D-1) and (D-2).

Now, for ϕ_m so large (λ so small) that $\phi_m/N \gg 1$, Equation (49) becomes [using Equations (47) and (48)]

$$\frac{(\delta I)_{rms}}{\langle I \rangle} \rightarrow \frac{\left[1 + \frac{1}{16}\sin^4(\phi_m/2)/\cos^4(\phi_m/2)\right]^{1/2}}{1 + \frac{1}{4}\sin^2(\phi_m/2)/\cos^2(\phi_m/2)} , \tag{D-3}$$

which oscillates rapidly with λ and is bounded by 1 and $\sqrt{17}/5$. If a spread in ϕ_m exists, then $\cos(\phi_m/2) \rightarrow 0$ for large ϕ_m (or small λ) and $(\delta I)_{rms}/\langle I \rangle \rightarrow 1$ with no oscillations.

Whenever $\lambda \rightarrow \infty$, then $\phi_m \rightarrow 0$ and, hence, $(\delta I)_{rms}/\langle I \rangle \rightarrow 0$. Thus, for $\lambda > \lambda_1$, $(\delta I)_{rms}/\langle I \rangle$ falls off from unity and approaches zero.

The sketch illustrating these mathematical points in shown in Figure 4.

LIST OF SYMBOLS

A	Far-field electric field amplitude
ΔA	Variation of A from the average A_a
$a_o;a$	Electric field amplitude of light incident on (reflected from) the constructed surface of the target
a_1	Electric field amplitude of reflected light at constructed surface without phase factor due to curvature
Δa_1	Variation of a_1 from the average a_{1a}
B	Constant for given wavelength and surface shape and size
F	Factor which depends only on constructed surface size and shape at any point
g	Constant-of-proportionality between I and A^*A
H	Distance between actual target surface and the constructed surface along the path of reflected light
h	Deviation of surface "height" from the constructed surface (for which $<\phi> = 0$)
h_m	Maximum value of the cell-averaged-h over the exposed target surface
I	Far-field light intensity upon reflection
ΔI	Variation of I from the average $<I>$
Im	Imaginary part of —
k	Magnitude of light wave vector
ℓ_c	Average decorrelation length $<\ell_i>$ based on autocorrelation of Δa_1
ℓ_{pv}	"Peak-to-valley" distance for Δa_1 at small ϕ_m
N	Number of decorrelation area cells on exposed target surface
$\underline{n}$	Surface normal vector
p	Reflectance of target surface

Δp	Path length difference between incident and reflected light at constructed surface
R	Distance from observer to nearest point on target
r	Distance from observer to a point on the constructed surface
Re	Real part of —
S	Exposed surface area of target
S_a	Cross-section area of target
α	Surface orientation angle for target
Γ	Ratio $(\delta I)_{rms}/\langle I \rangle$
δ	Fluctuation of —
δ_i'	Range factor for variation of ϕ_{mi}
θ	Angle between $\underline{r}$ and $\underline{n}$ (for large distances)
ϕ	Phase change upon reflection due to detailed surface property fluctuations
ϕ_m	Maximum value of ϕ over the exposed target surface
ψ	Angle between light reflected through the constructed surface and the line from that point to observer

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